Python程序设计

函数 - 基础知识与递归

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本节涉及到的知识点

- 函数的定义和使用
- 变量的作用域和参数传递
- 高阶函数
- 函数嵌套
- 递归

创建和使用函数

- def语句创建一个函数对象并将其赋值给一个变量

```
1 def intersect(A, B):
2     res = []
3     for x in A:
4         if x in B:
5             res.append(x)
6     return res
```

  - 执行def语句之后，创建了一个函数对象，并将变量intersect绑定到该对象　简单起见，直接称呼函数intersect

- 在交互模式中使用函数：函数名(参数)

```
>>> C = intersect(LA, LB)
```

def也是复合语句，所以首行末尾需要冒号，其后的代码块需要缩进

函数调用

- 当函数调用时，调用者暂停执行，直到该函数遇到**return exp**语句执行结束，并返回表达式**exp**的值

```
1 def intersect(A, B):
2     res = []
3     for x in A:
4         if x in B:
5             res.append(x)
6     return res
```

- 调用者获得该返回值，并继续程序的执行

```
>>> LA = [1, 2, 3, 4, 5]
>>> LB = [2, 3, 5, 7, 11]
>>> C = intersect(LA, LB)
>>>
```

如何使用列表推导计算两个列表的交集?

return语句

- return语句结束函数调用，并且：
  - 如果return语句包含一个表达式，则返回该表达式的值
  - 如果return语句不包含表达式，则返回一个None对象
- 如果函数没有return语句，则执行完其内部所有语句后结束函数调用，并自动返回一个None对象

```
>>> def g():
        x = 1
        return x

>>> print(g())
1
```

```
>>> def h():
        x = 1
        if x: return
        return x

>>> print(h())
None
```

```
>>> def f():
        x = 1

>>> print(f())
None
```

考拉兹猜想

- 对于任何一个正整数，如果它是奇数，对它乘3再加1，如果它是偶数，对它除以2，如此循环，最终都能够得到1

考拉兹序列 13 40 20 10 5 16 8 4 2 1

- 编写一个函数collatz，具有一个名为n的参数
  - 如果n是奇数，返回3*n+1
  - 如果n是偶数，返回n//2
- 编写一个函数trace_collatz，接受一个正整数，以列表形式返回其对应的考拉兹序列

考拉兹猜想

```python
def collatz(n):
    if n % 2 == 1:
        return 3 * n + 1
    else:
        return n // 2

def trace_collatz(n):
    res = [n]
    while True:
        n = collatz(n)
        res.append(n)
        if n == 1:
            break
    return res
```

```
>>> trace_collatz(1)
[1, 4, 2, 1]
>>> trace_collatz(2)
[2, 1]
>>> trace_collatz(3)
[3, 10, 5, 16, 8, 4, 2, 1]
```

变量的作用域

- 在函数内部赋值的变量只能在函数内部使用

```
>>> def intersect(A, B):
        res = []
        for x in A:
            if x in B:
                res.append(x)
        return res

>>> res
Traceback (most recent call last):
  File "<pyshell#16>", line 1, in <module>
    res
NameError: name 'res' is not defined
```

变量的作用域

- 在函数内部赋值的变量和在函数外部赋值的变量即使重名，也是两个不同的变量

```
>>> def intersect(A, B):
        res = []
        for x in A:
            if x in B:
                res.append(x)
    return res

>>> res = 'outside variable res'
>>> intersect([1, 2, 3], [2, 3, 5])
[2, 3]
>>> res
'outside variable res'
```

这是两个不同的变量

参数传递

- 调用函数的时候需要提供实际参数
  - 不可变类型（数值、字符串等）：形参获得实参的值
  - 可变类型（列表等）：形参获得实参的引用

```
>>> def f(a, b):
        a = 8
        b[0] = 10


>>> x = 1
>>> y = [1, 3, 5]
>>> f(x, y)
```

```
>>> x
1
>>> y
[10, 3, 5]
```

line that has just executed

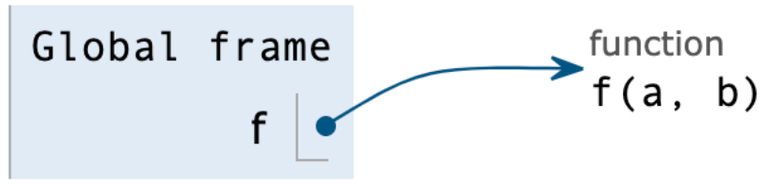
next line to execute

```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```

Frames Objects

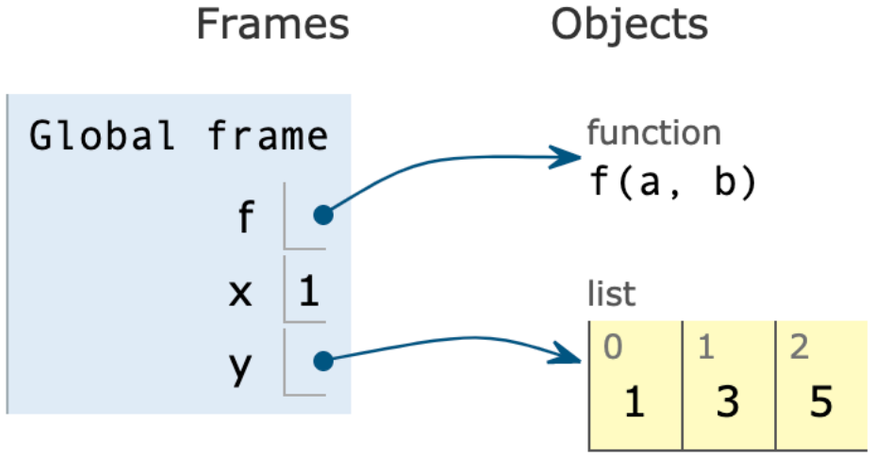
```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```

Frames Objects




```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```

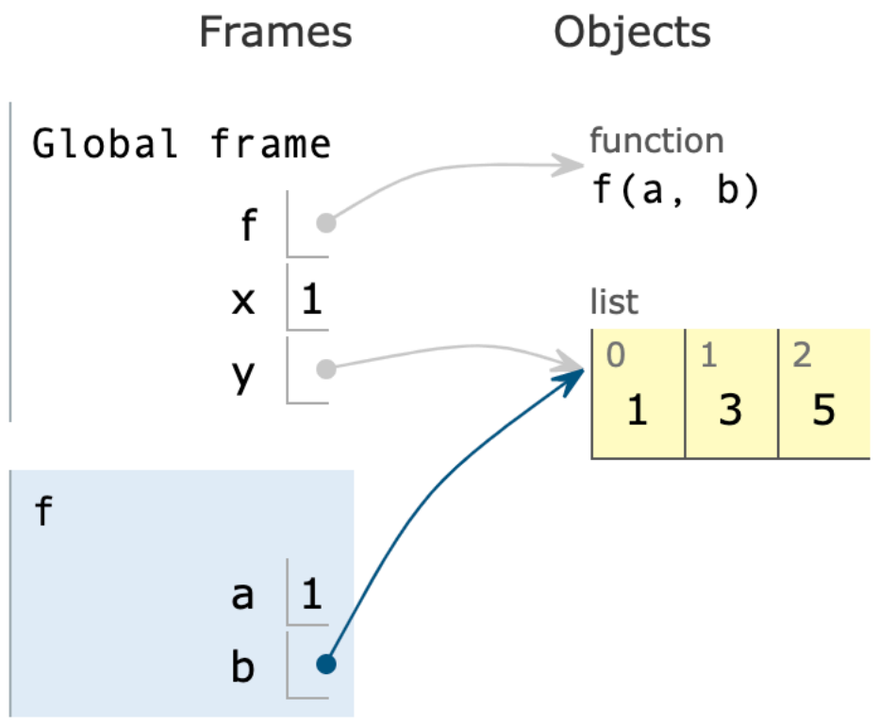
Frames Objects



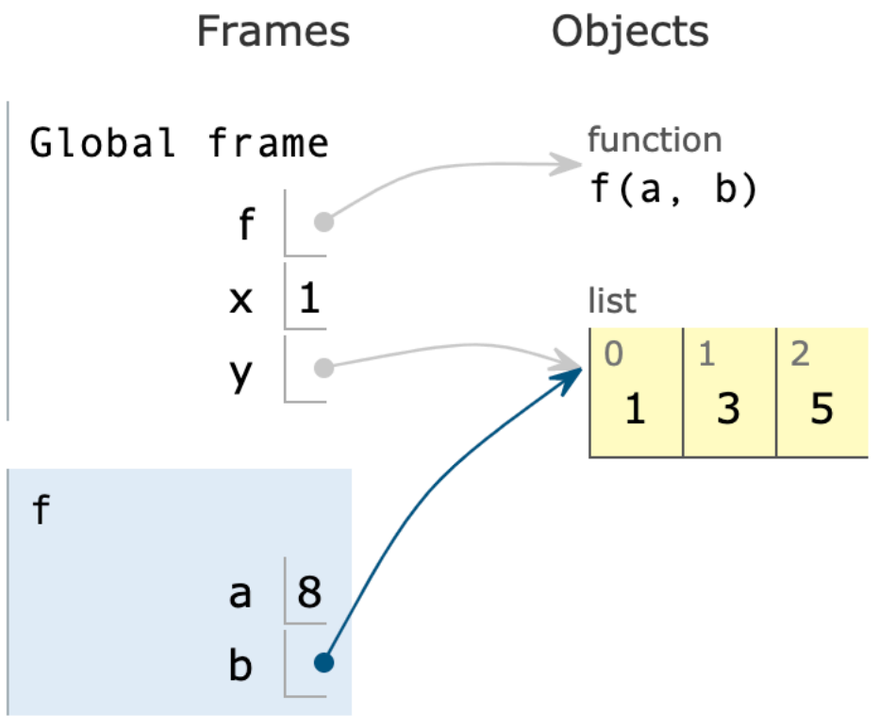
line that has just executed

next line to execute

```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```




```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```

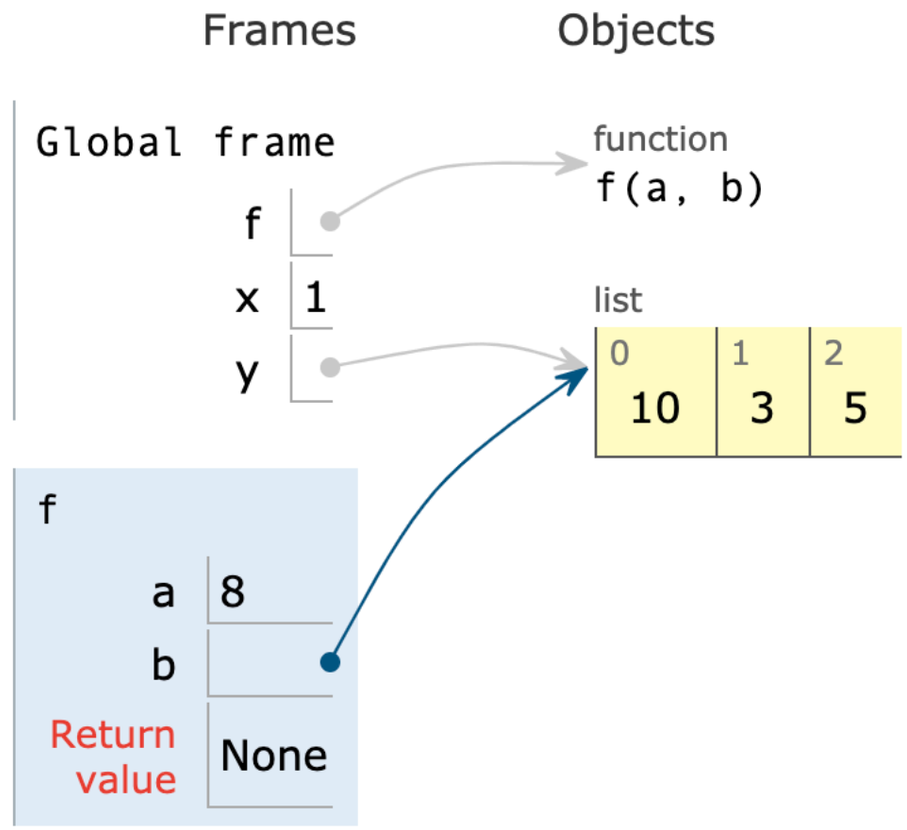


✍ 上图通过代码可视化工具生成 - http://www.pythontutor.com/

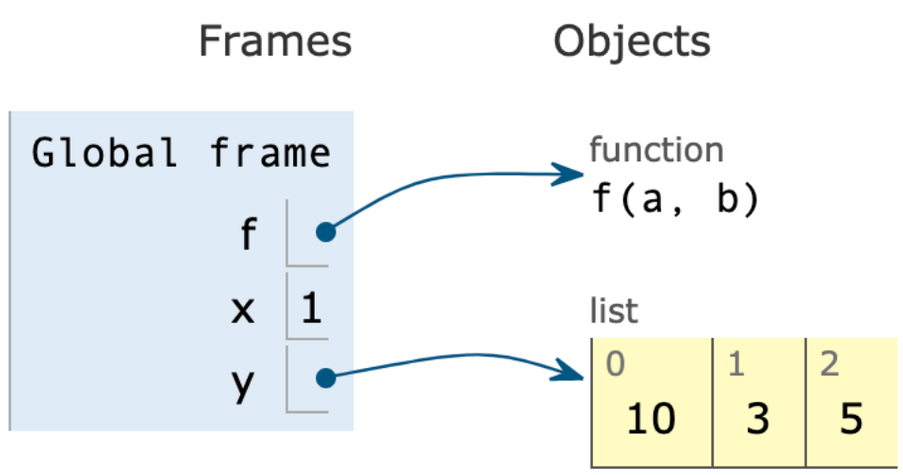
line that has just executed

next line to execute

```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```




```
1  def f(a, b):
2      a = 8
3      b[0] = 10
4
5  x = 1
6  y = [1, 3, 5]
7  f(x, y)
```



✍ 上图通过代码可视化工具生成 - http://www.pythontutor.com/

函数是对象

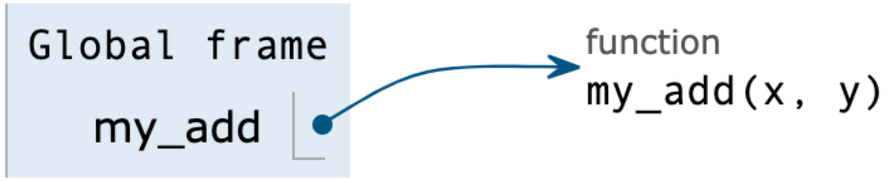
- 函数也是对象，所以可以
  - 把函数分配给变量
  - 把函数存储在数据结构（比如列表、字典）中
  - 作为参数传递给其他函数
  - 作为其他函数的返回值

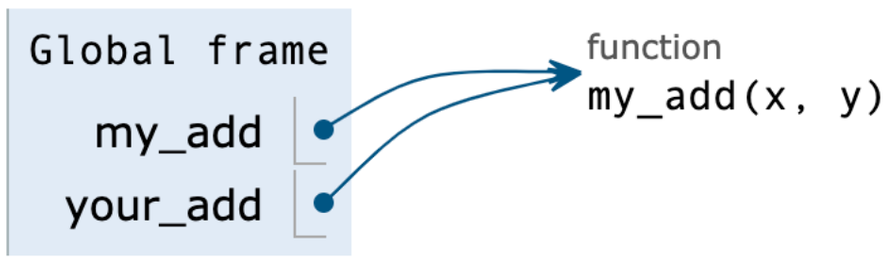
把函数分配给变量

- def语句创建一个函数对象并将其赋值给一个变量
- 赋值语句可以将多个变量绑定到同一个函数

```
>>> def my_add(x, y):
        return x + y
```

```
>>> my_add(3, 5)
8
>>> your_add = my_add
>>> your_add(1, 10)
11
```





函数存储在数据结构中

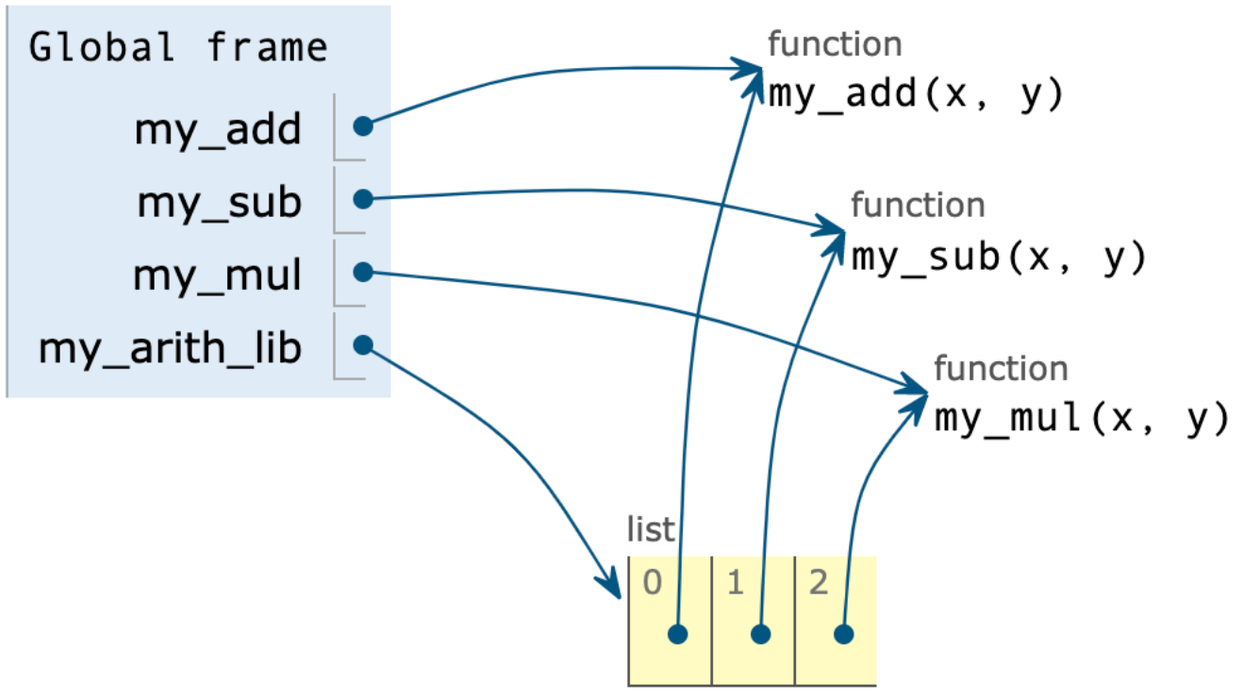
- 列表是一组任意类型的对象，所以函数也能存放其中

```
>>> def my_sub(x, y):
        return x - y

>>> def my_mul(x, y):
        return x * y

>>> my_arith_lib = [my_add,
                    my_sub,
                    my_mul,
                   ]
>>> for f in my_arith_lib:
        print(f(3, 5), end = ' ')


8 -2 15
```



函数作为其他函数的参数

- 不同的参数（传递进来的函数）导致函数具有不同的行为

```
>>> def calculate(f, x, y):
        return f(x, y)

>>> calculate(my_add, 3, 5)
8
>>> calculate(my_mul, 3, 5)
15
```

- 接受其他函数作为参数的函数称为高阶函数

- map(function, iterable, …)：返回一个可迭代对象，其中的元素按照下面的规则生成：依次将iterable中的元素作为参数来调用函数function得到的值的序列

高阶函数map

- map(function, iterable, …)：返回一个可迭代对象，其中的元素按照下面的规则生成：依次将iterable中的元素作为参数来调用函数function得到的值的序列

```
>>> A = [-2, -1, 0, 1, 2]
>>> B = [1, 2, 3, 4, 5]
>>>
>>> list(map(abs, A)) # 内置函数abs接受一个参数
[2, 1, 0, 1, 2]
>>>
>>> list(map(my_add, A, B)) # my_add接受两个参数
[-1, 1, 3, 5, 7]
```

函数可以嵌套

- 在函数中可以通过def语句定义另一个函数

```
>>> def maker(n): # 外层函数
        def action(x): # 内层函数
            return x ** n
        return action # 返回函数action，该函数记住了n的值

>>> f = maker(2) #调用函数maker，创建函数action，n的值为2
>>>
>>> f(3) # 调用函数action，参数x为3，n的值为2
9
>>> f(4) # 调用函数action，参数x为4，n的值为2
16
>>> g = maker(3) # 创建函数action的另一个版本，n的值为3
>>>
>>> g(3) # 调用函数action的另一个版本，n的值为3
27
```

递归

- 直接或者间接调用自己的函数
- 计算正整数n的阶乘：$n! = n \times \overbrace{(n-1) \times (n-2) \cdots \times 2 \times 1}^{(n-1)!}$
- 如果函数$f(n)$返回n的阶乘，那么$f(n) = n \times f(n-1)$
- 递归的两种情况
  - 递归情况：调用自己解决一个类似但规模较小的问题
    - 要计算$f(10)$，只需计算$f(9)$，然后返回$10 \times f(9)$
  - 基本情况：直接可以得到解，无需再次调用自己
    - 要计算$f(1)$，根据定义知道1的阶乘是1

递归

```
>>> def my_fac(n):
        if n == 1:
            return 1
        else:
            return n * my_fac(n-1)
```

```
>>> my_fac(5)
120
>>> my_fac(20)
2432902008176640000
>>> import math
>>> math.factorial(20)
2432902008176640000
```

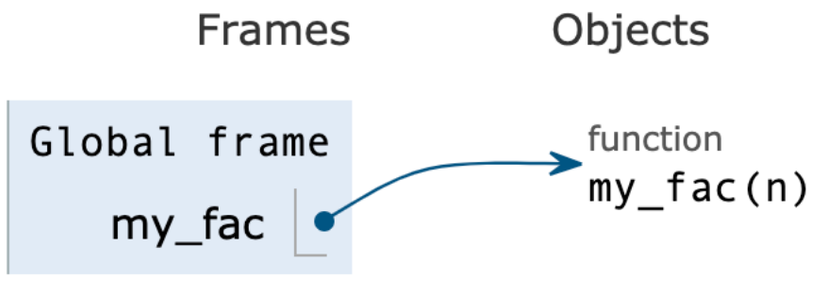
✍️ **如果调用my_fac函数的参数是0或者-1，情况会如何?**

line that has just executed

next line to execute

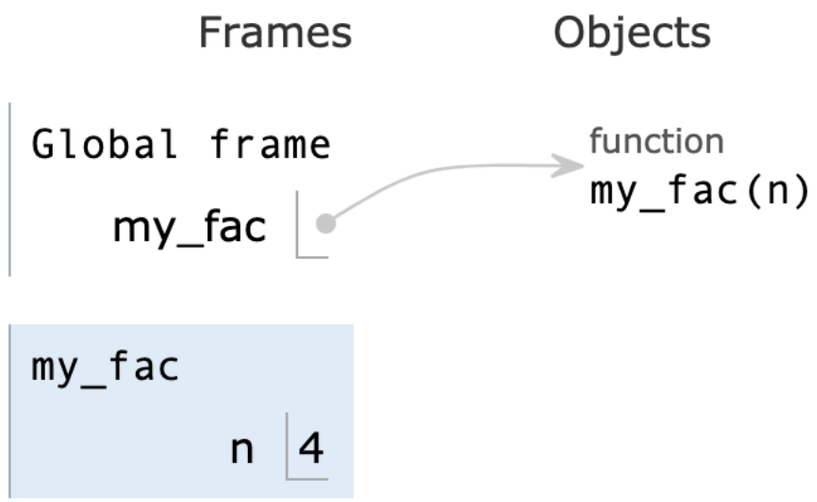
(1)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



(2)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



(3)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

(4)

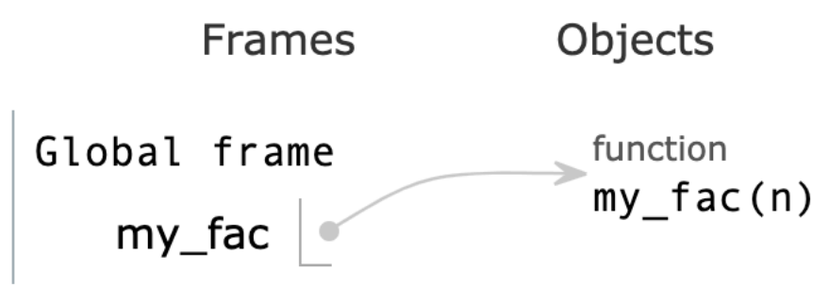
```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

line that has just executed

next line to execute

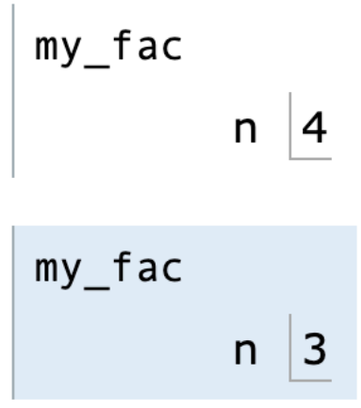
(5)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



(6)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



(7)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

line that has just executed

next line to execute

(8)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

Frames Objects

Global frame

my_fac

function
my_fac(n)

(9)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

my_fac

n | 4

(10)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

my_fac

n | 3

my_fac

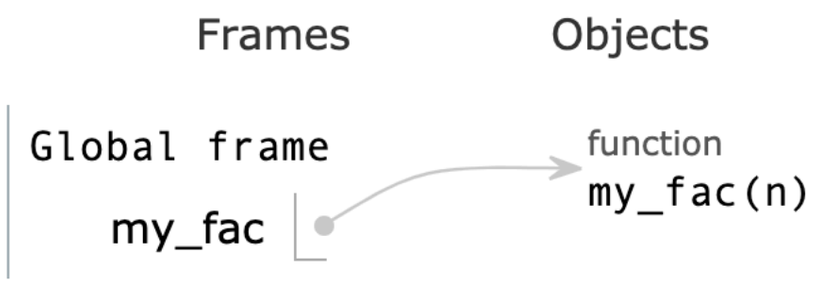
n | 2

line that has just executed

next line to execute

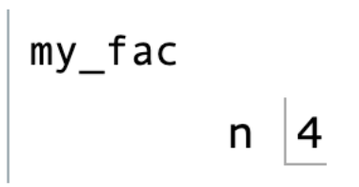
(11)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



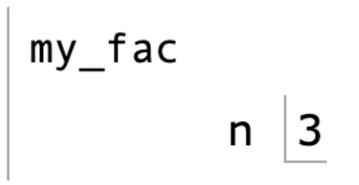
(12)

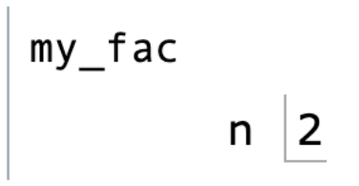
```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

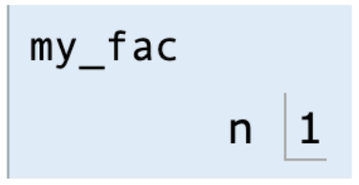


(13)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```

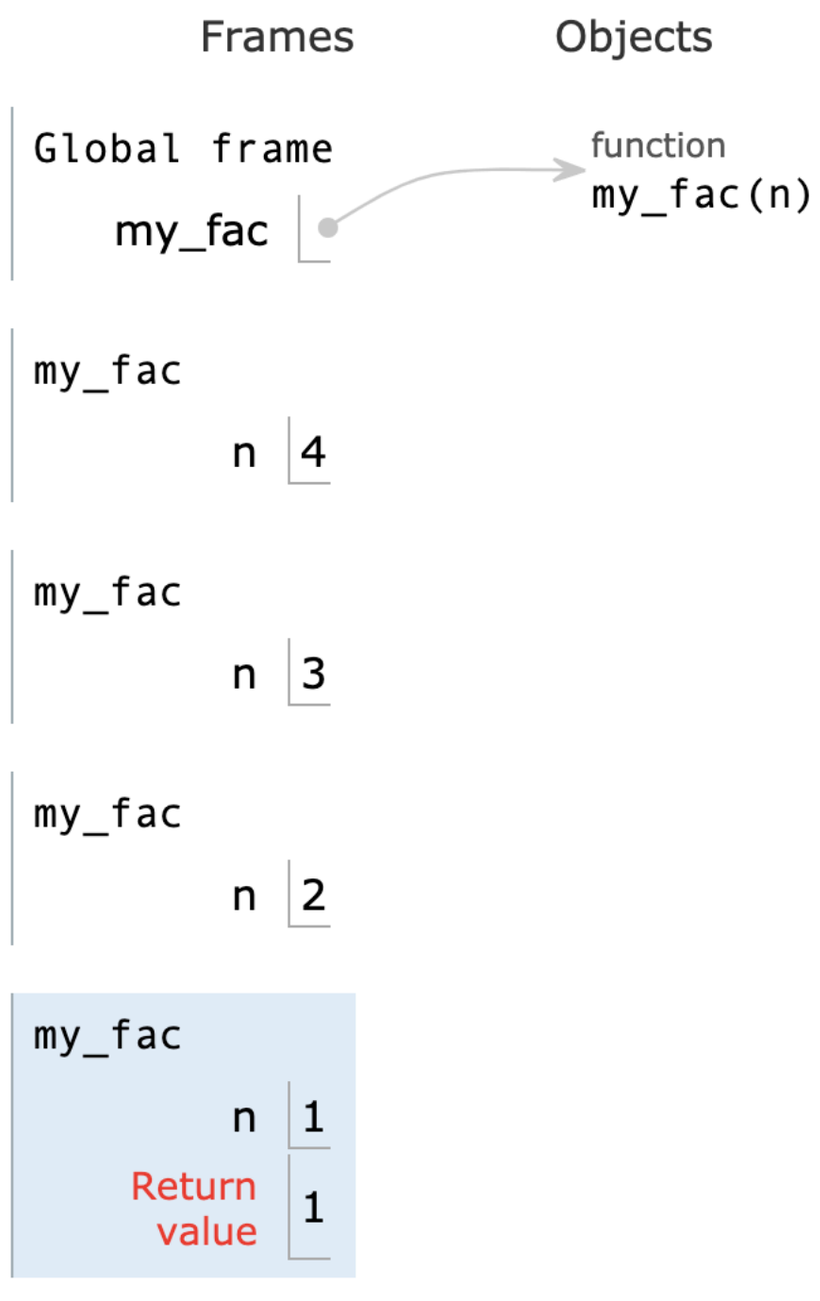






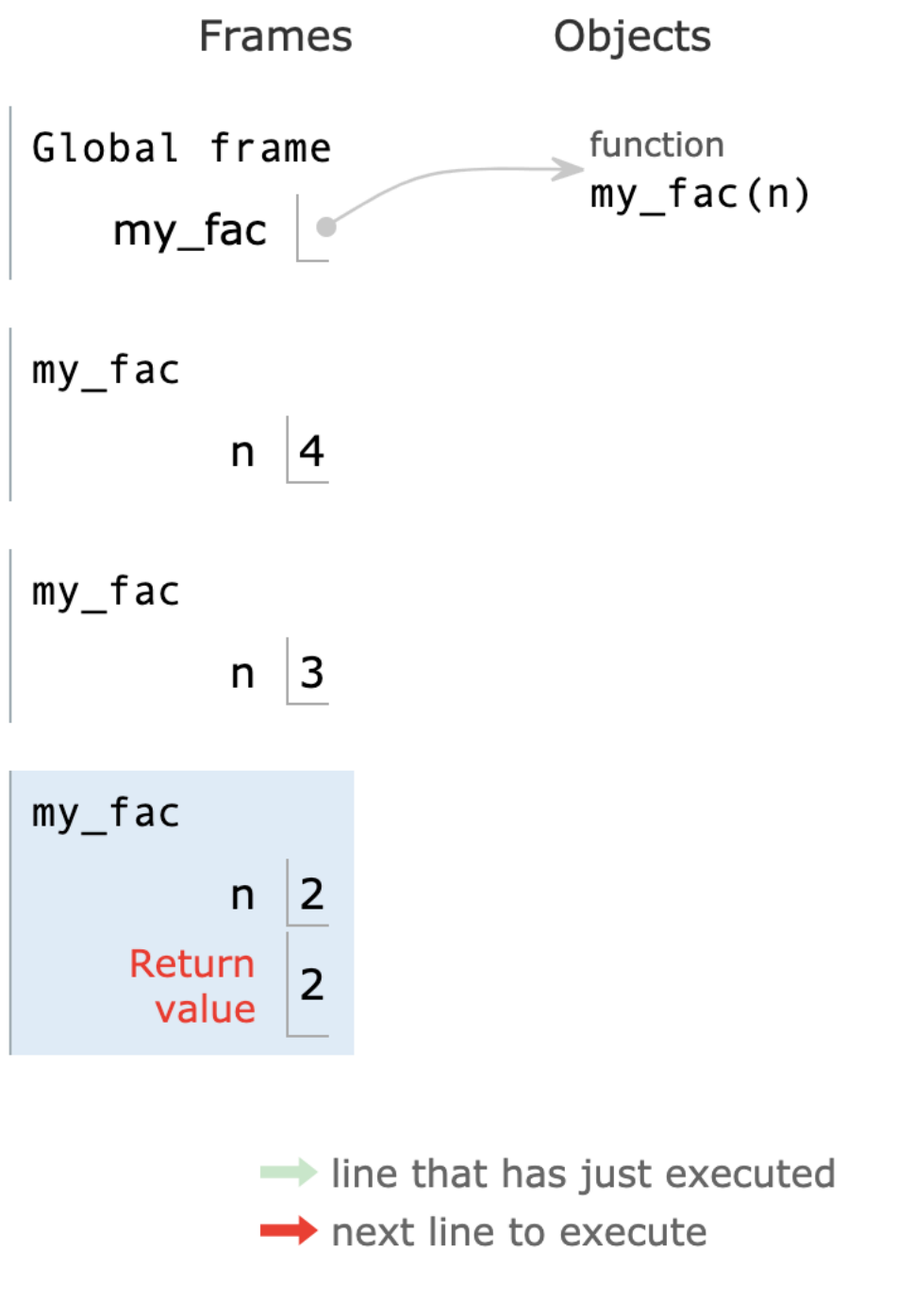
(14)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



(15)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



line that has just executed

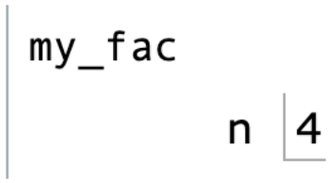
next line to execute

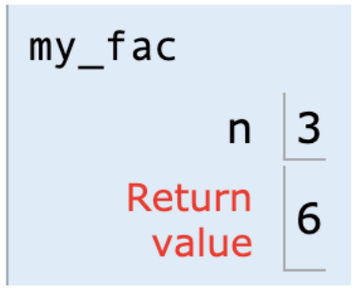
line that has just executed

next line to execute

(16)

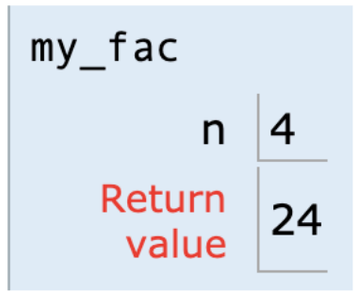
```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```





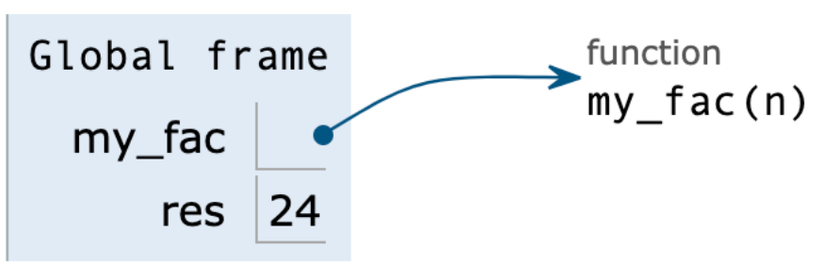
(17)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



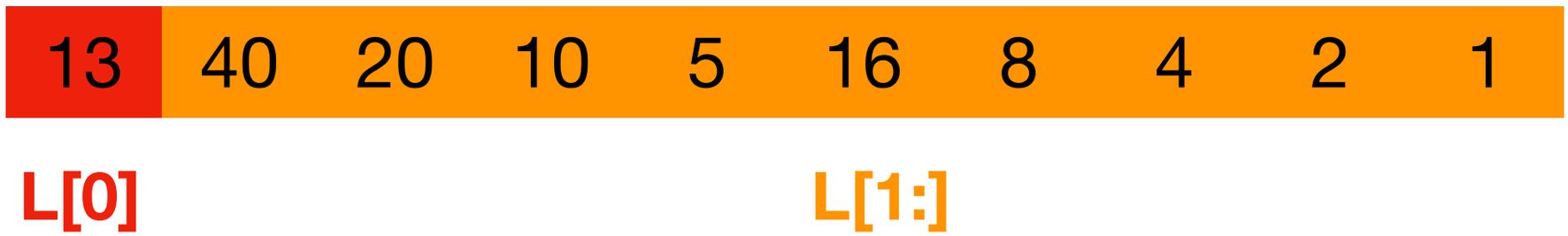
(18)

```
1  def my_fac(n):
2      if n == 1:
3          return 1
4      else:
5          return n * my_fac(n-1)
6
7  res = my_fac(4)
```



列表求和

- 编写一个函数my_sum，接受一个列表L，求所有元素之和
- 要求：不使用内置sum函数，使用递归
- 基本情况 - 列表为空，返回0
- 递归情况 - 列表不为空，返回L[0]+my_sum(L[1:])

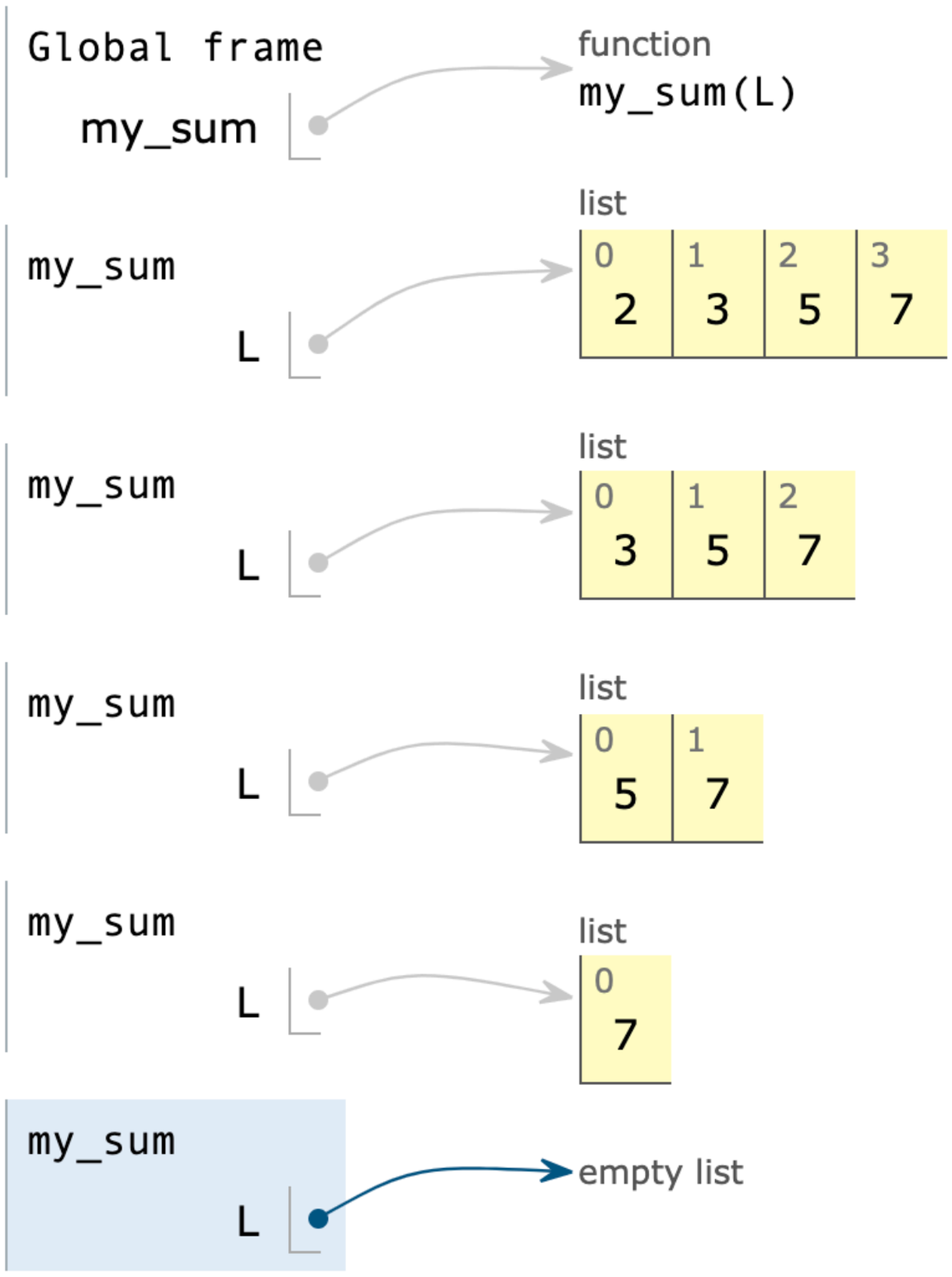


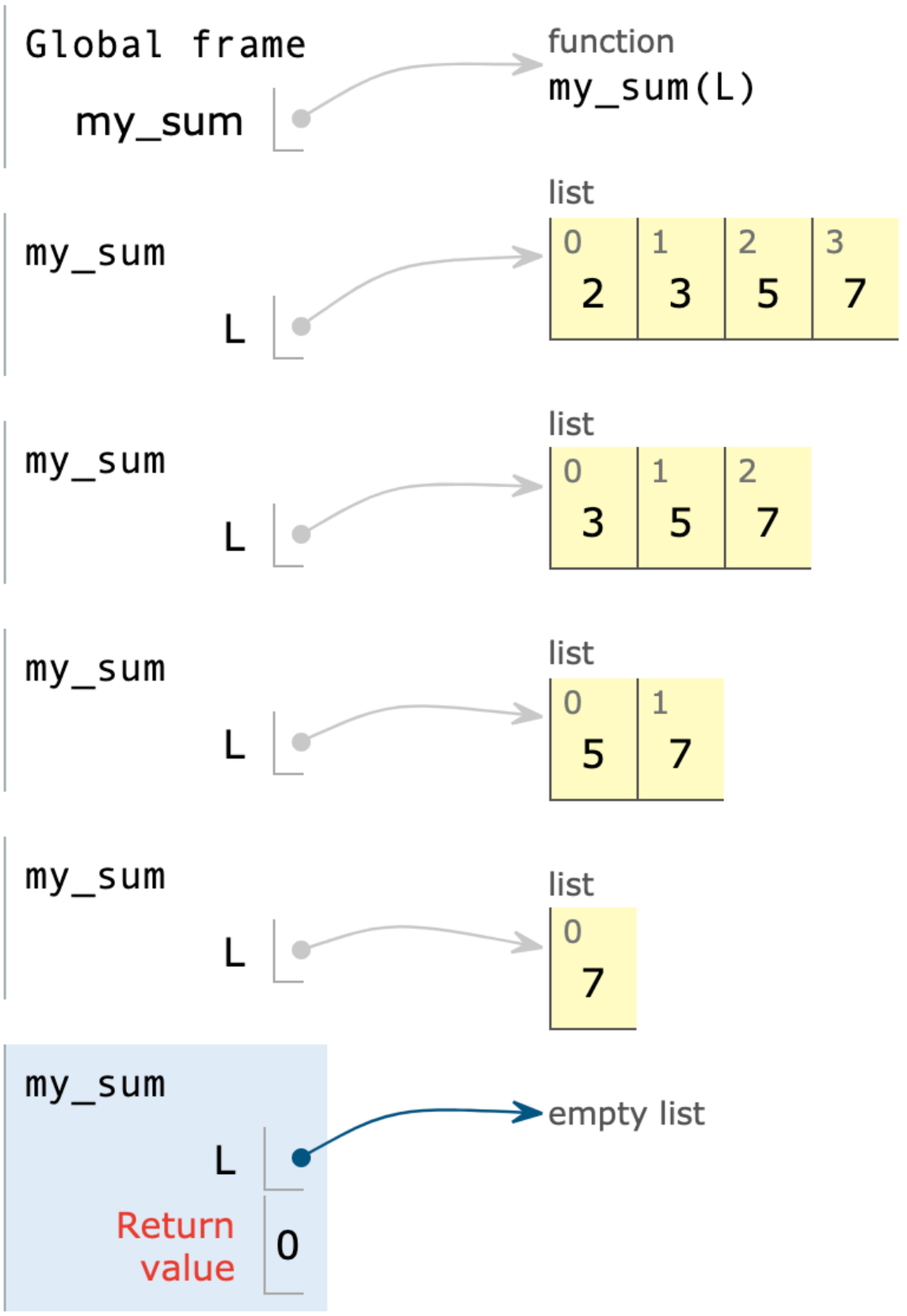
用递归求和

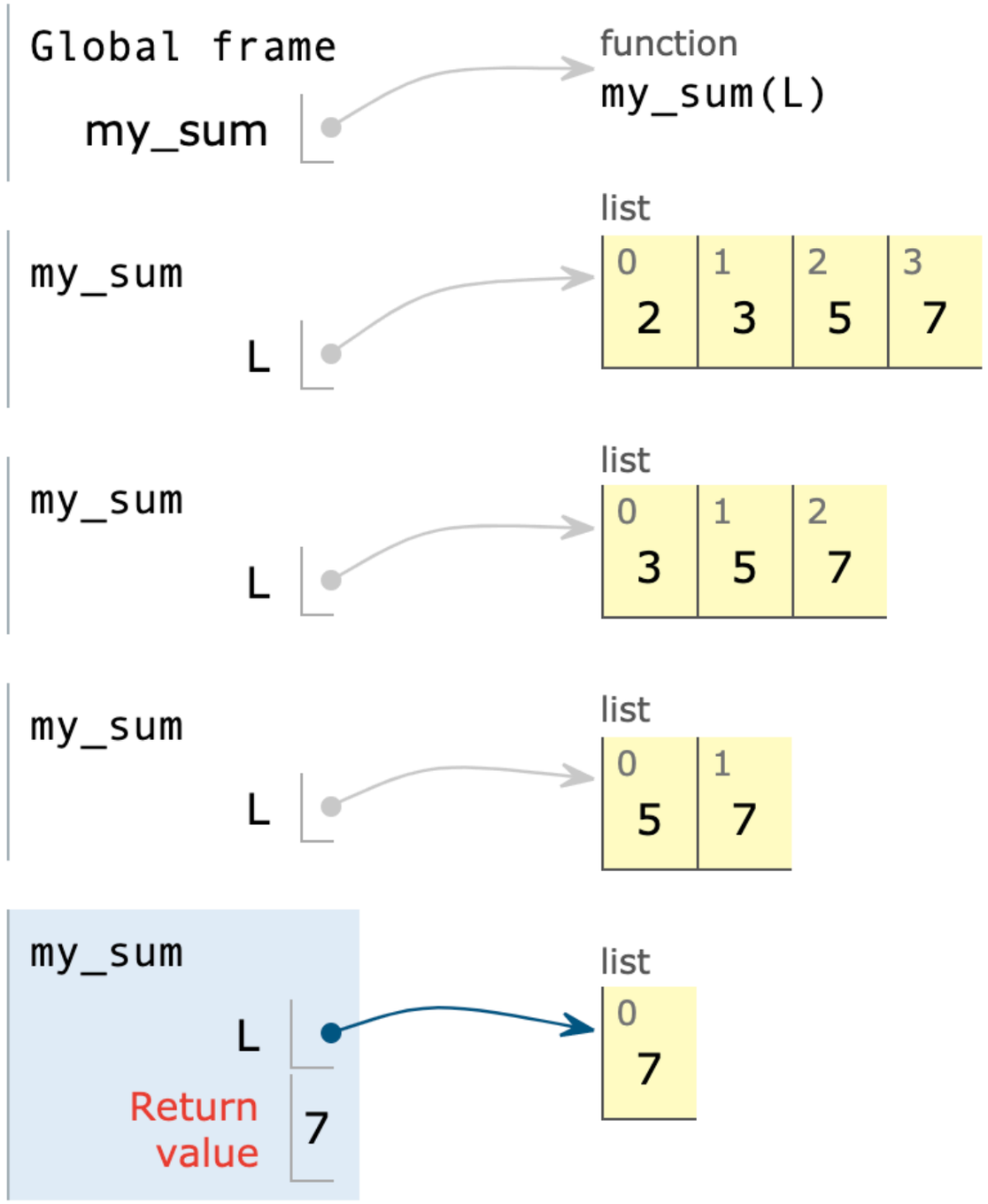
- 编写一个函数my_sum，接受一个列表L，用递归求和
- 基本情况 - 列表为空，返回0
- 递归情况 - 列表不为空，返回L[0]+my_sum(L[1:])

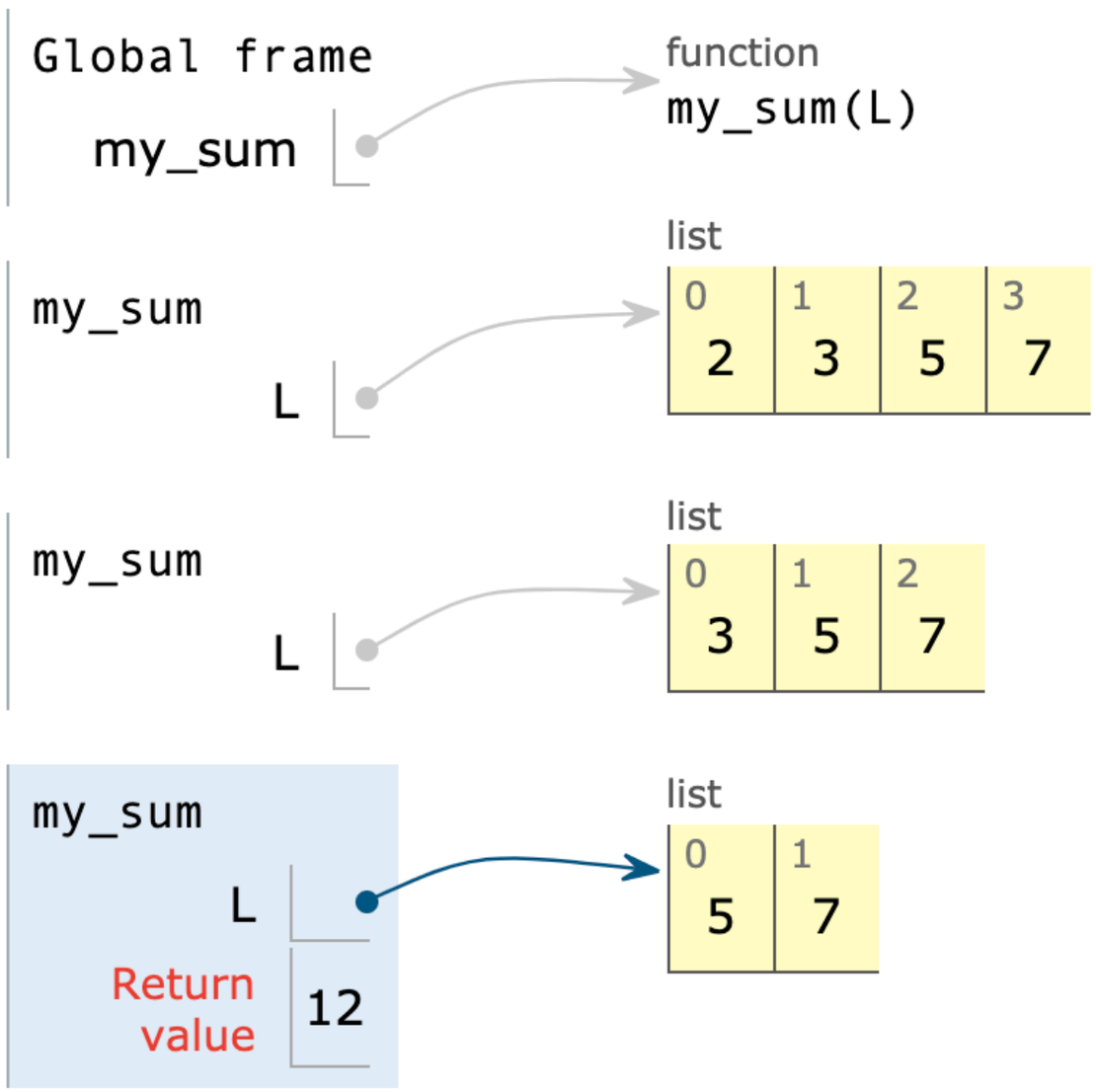
```
def my_sum(L):
    if not L:
        return 0
    else:
        return L[0] + my_sum(L[1:])
```

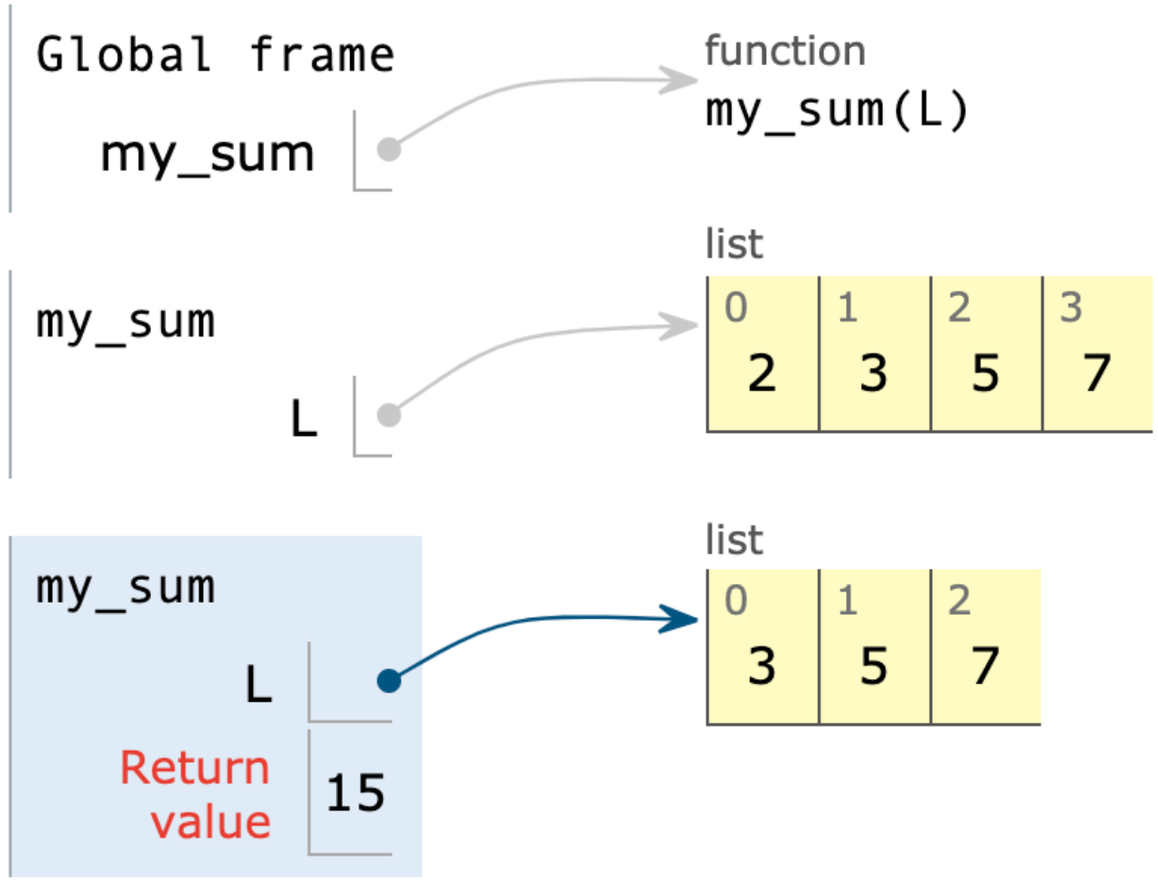
```
>>> res = my_sum([2, 3, 5, 7])
>>> res
17
```

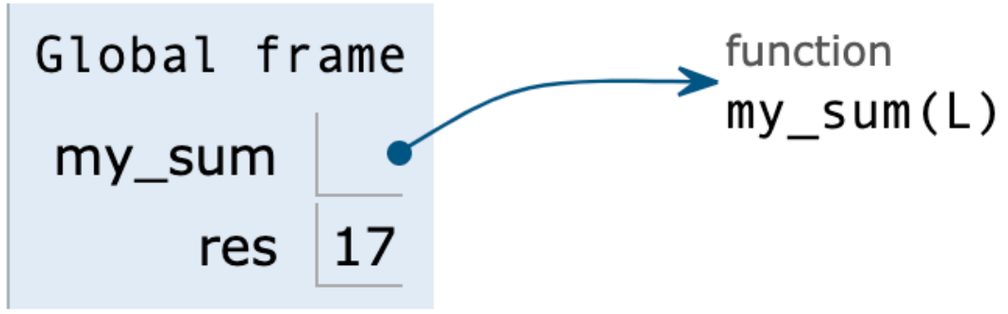

Global frame
my_sum
function
my_sum(L)
my_sum
L
list
0
2
1
3
2
5
3
7
my_sum
L
list
0
3
1
5
2
7
my_sum
L
list
0
5
1
7
my_sum
L
list
0
7
my_sum
L
empty list

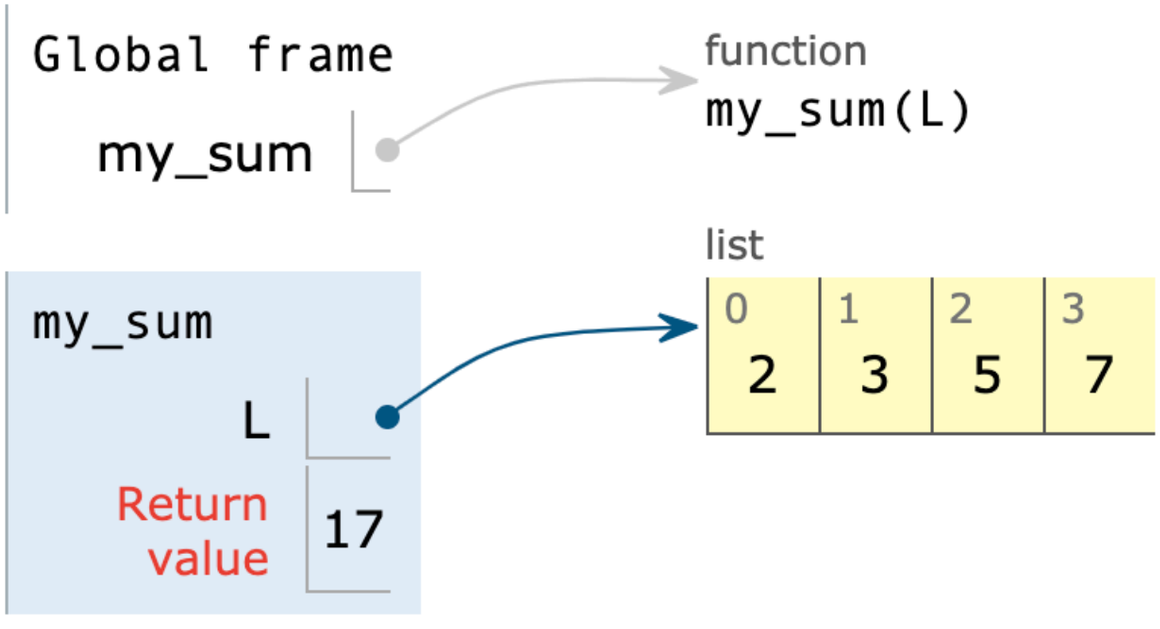

Global frame
my_sum
function
my_sum(L)
my_sum
L
list
0
2
1
3
2
5
3
7
my_sum
L
list
0
3
1
5
2
7
my_sum
L
list
0
5
1
7
my_sum
L
list
0
7
my_sum
L
Return value
0
empty list


Global frame
my_sum
function
my_sum(L)
my_sum
L
list
0
2
1
3
2
5
3
7
my_sum
L
list
0
3
1
5
2
7
my_sum
L
list
0
5
1
7
my_sum
L
Return value
7
list
0
7


Global frame
my_sum
function
my_sum(L)
my_sum
L
list
0
2
1
3
2
5
3
7
my_sum
L
list
0
3
1
5
2
7
my_sum
L
Return value
12
list
0
5
1
7


Global frame
my_sum
function
my_sum(L)
my_sum
L
list
0
2
1
3
2
5
3
7
my_sum
L
Return value
15
list
0
3
1
5
2
7


Global frame
my_sum
res
17
function
my_sum(L)


Global frame
my_sum
function
my_sum(L)
my_sum
L
Return value
17
list
0
2
1
3
2
5
3
7

嵌套列表求和

- 内置sum函数可以对一个列表求和
- 注意：如果列表的元素不能相加，则会报错

```
>>> L = [2, 3, 5, 7]
>>> sum(L)
17
>>> A = [1, [2, [3, 4], 5], 6, [7, 8]]
>>> sum(A)
Traceback (most recent call last):
  File "<pyshell#69>", line 1, in <module>
    sum(A)
TypeError: unsupported operand type(s) for +:
'int' and 'list'
```

嵌套列表求和

- 编写一个函数my_sum_v1，接受一个嵌套列表（嵌套深度随意），求该列表中所有数值的和
- 基本情况 - 当前元素是一个数值，直接返回
- 递归情况 - 当前元素是一个列表
  - 对于列表中的每一个元素，使用my_sum_v1求和
  - 遍历完成之后，返回总的和
- 如何判断一个元素是否是列表？
  - 使用内置函数isinstance(obj, classinfo)进行判断

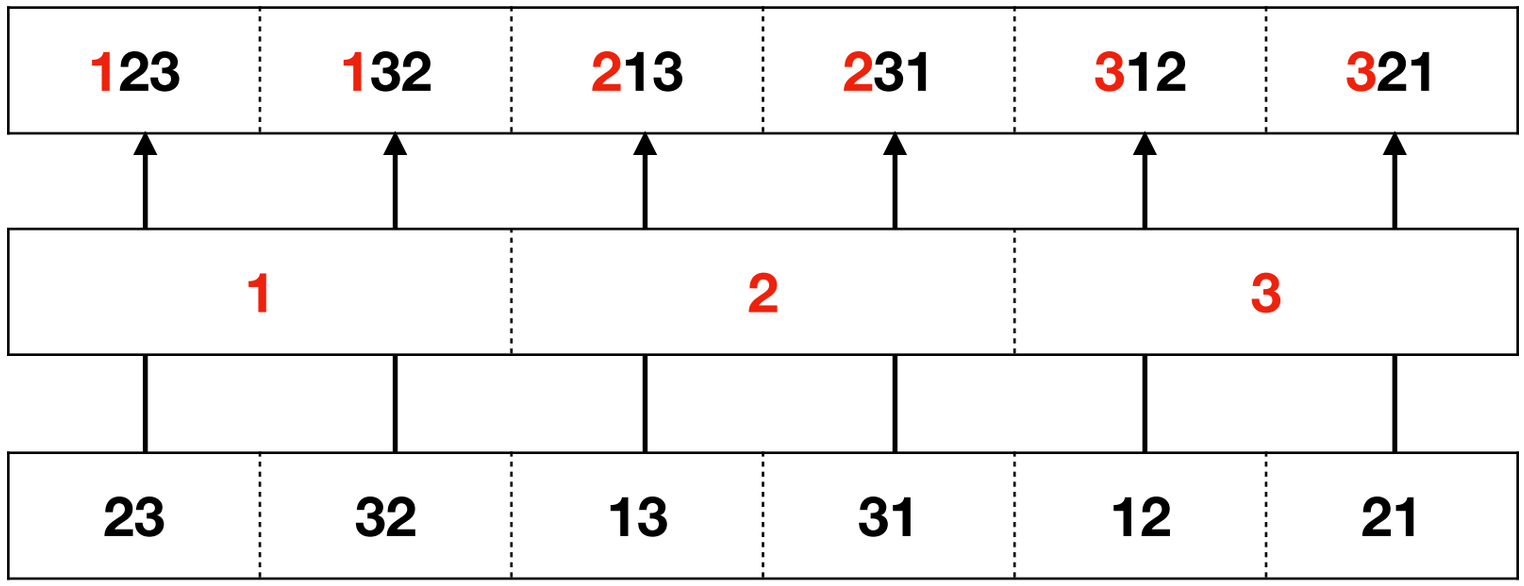
嵌套列表求和

- 基本情况 - 当前元素是一个数值，直接返回
- 递归情况 - 当前元素是一个列表
  - 对于列表中的每一个元素，使用my_sum_v1求和
  - 遍历完成之后，返回总的和

```
def my_sum_v1(L):
    if not isinstance(L, list):
        return L
    else:
        total = 0
        for x in L:
            total = total + my_sum_v1(x)
        return total
```

全排列

- 给定n个元素，生成它们的全排列。比如元素1,2,3的全排列共有6种情况：123、132、213、231、312、321
- 2个元素a和b的全排列只有两种情况：ab和ba
- 3个元素的全排列如何由2个元素的全排列得到？



全排列

- 给定n个元素，生成它们的全排列。比如元素1,2,3的全排列共有6种情况：123、132、213、231、312、321
- 假设函数permute(L)可以生成列表L中所有元素的全排列
- n个元素的全排列如何生成？
  - 任一元素 + 剩余n-1个元素的全排列
    - L[0] + permute(L[1:])
    - L[i] + permute(L[:i] + L[i+1:])
    - L[n-1] + permute(L[:n-1])

全排列

```
>>> def permute(L):
        if not L:
            return [L] # 应该返回[[]]
        else:
            res = []
            for i in range(len(L)):
                rest = L[:i] + L[i+1:]
                # 如果rest为空，permute函数应该返回[[]]
                for x in permute(rest):
                    res.append(L[i:i+1] + x)
        return res


>>> permute([1,2,3])
[[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]]
>>> len(permute([1,2,3,4,5]))
120
```